

# The Standard Model as an Open-System Effective Theory: Latency Portals, Running Dissipation, and Precision-Spectroscopy Kill-Tests (MetaTime v40.3: Derived Line-Shape Constraints and Portal-RG Consistency)

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We formulate a minimal open-system completion of the MetaTime program in which the observed Standard Model (SM) dynamics arises as a reduced boundary description interacting with unobserved bulk degrees of freedom. The extension is encoded by a causal non-Markovian influence functional on the Schwinger–Keldysh closed-time path. The central control parameter is a dimensionless *latency/impedance* amplitude  $\Gamma_L(\mu)$  that measures dissipative strength relative to a characteristic boundary scale  $\mu$ . In contrast to earlier drafts, we (i) enforce portal–renormalization consistency by adopting a minimal *safe irrelevant* SM-singlet latency portal of scaling dimension  $\Delta = 6$  and deriving the corresponding running exponent  $\eta \simeq 2(\Delta - 4) = 4$ , while showing how electroweak symmetry breaking produces an effective trace-coupling in the infrared; and (ii) derive an explicit precision-spectroscopy constraint from the Markov/Lindblad limit of the influence functional: the latency-induced homogeneous linewidth for an atomic transition  $n \leftrightarrow m$  obeys  $\Delta\nu_{nm} \simeq \frac{\gamma}{2\pi} (\Delta L_{nm})^2$  with  $\gamma = \Gamma_L(\mu)\mu$ , where  $\Delta L_{nm}$  is a portal matrix-element difference. Applying this to hydrogen 1S–2S yields a falsifiable bound on  $\Gamma_L(\mu_H)$  given a matching scale  $\Lambda_L$ . Finally, we connect the proton-persistence barrier  $S_{\text{eff}}$  (MetaTime “Anchor” conjecture) to a conservative microscopic envelope for  $\Gamma_L$  via a coarse-grained multi-channel model with explicit  $N_{\text{eff}}$  dependence, and identify muonic and highly charged hydrogen-like systems as amplification targets.

## I. MOTIVATION

The SM is an extraordinarily successful local quantum field theory, but it is not, by itself, a theory of *why* the observed low-energy dynamics is so robust against environmental disturbances and why macroscopic persistence is so stable over cosmological times. MetaTime proposes that “inertia” and “persistence” can be reinterpreted as costs in a finite-resource holographic processing manifold. Operationally, this motivates describing the observable 4D world as a boundary subsystem interacting with unobserved bulk degrees of freedom, i.e. an *open quantum system*.

Any open-system completion must pass precision low-energy constraints. This revision (v40.3) addresses the key referee vulnerability: earlier drafts used a dimensional proxy for spectroscopy. Here we derive the relevant line-shape corrections from the reduced dynamics implied by the influence functional, and we make the portal–running structure self-consistent.

We work in units  $\hbar = c = 1$  unless otherwise stated; frequencies are related to energies by  $E = 2\pi\nu$ .

## II. MINIMAL OPEN-SYSTEM EFT: CTP INFLUENCE FUNCTIONAL

We treat the observed 4D SM fields  $\phi$  as boundary degrees of freedom interacting with bulk/environmental variables  $\chi$ . The reduced dynamics is naturally described on the closed-time path (CTP) with doubled fields  $\phi_{\pm}$  [1–4]. Integrating out  $\chi$  yields an influence functional  $F[\phi_+, \phi_-] = \exp(iS_{\text{IF}})$ .

### A. A safe irrelevant latency portal ( $\Delta = 6$ ) and its IR reduction

A central constraint is gauge invariance and the absence of immediate conflict with established precision limits. We adopt a minimal *safe irrelevant* SM-singlet portal of scaling dimension  $\Delta = 6$ :

$$\mathcal{O}_L(x) \equiv \frac{1}{\Lambda_L^2} (H^\dagger H)(x) T^\mu{}_\mu(x), \quad (1)$$

where  $H$  is the Higgs doublet,  $T^\mu{}_\mu$  is the SM trace stress-energy, and  $\Lambda_L$  is a matching scale encoding bulk-to-boundary suppression. This operator is gauge invariant and does not introduce explicit baryon/lepton number violation at leading order.

After electroweak symmetry breaking,

$$H^\dagger H = \frac{(v+h)^2}{2} = \frac{v^2}{2} + vh + \frac{h^2}{2}, \quad (2)$$

so in the infrared (for processes not producing on-shell Higgs quanta) the portal reduces to an *effective* trace-coupling with coefficient  $(v^2/2\Lambda_L^2)$ :

$$\mathcal{O}_L(x) \rightarrow \frac{v^2}{2\Lambda_L^2} T^\mu{}_\mu(x) \quad (\text{IR, no Higgs excitation}). \quad (3)$$

This resolves the earlier inconsistency: the *fundamental* portal is irrelevant ( $\Delta = 6$ ), while its low-energy reduction reproduces a trace-like coupling with additional suppression.

## B. Quadratic influence action and kernels

At leading order in a Gaussian environment approximation, the latency influence action can be written as

$$S_{\text{IF}} = \frac{g_L^2}{2} \int d^4x d^4y \left[ \mathcal{O}_{L+}(x) - \mathcal{O}_{L-}(x) \right] K_R(x-y) \left[ \mathcal{O}_{L+}(y) + \mathcal{O}_{L-}(y) \right] \\ + \frac{ig_L^2}{2} \int d^4x d^4y \left[ \mathcal{O}_{L+}(x) - \mathcal{O}_{L-}(x) \right] K_N(x-y) \left[ \mathcal{O}_{L+}(y) - \mathcal{O}_{L-}(y) \right], \quad (4)$$

where  $g_L$  is the (dimensionful) portal coupling consistent with  $\Delta = 6$  (e.g.  $g_L \sim 1$  in units of  $\Lambda_L^{-2}$  after the normalization in Eq. (1)).  $K_R$  is causal/retarded and encodes dissipation/memory;  $K_N$  encodes noise.

A practical causal parameterization is the exponential memory kernel,

$$K_R(t, \mathbf{r}) = \Theta(t) \frac{1}{\tau} e^{-t/\tau} f\left(\frac{|\mathbf{r}|}{\ell}\right), \quad (5)$$

with memory time  $\tau$  and spatial coherence length  $\ell$ . The overall dissipative strength is carried by  $g_L^2$  together with the spectral normalization of  $K_R$ ; we package these into an effective rate  $\gamma$  below.

### C. Minimal fluctuation–dissipation consistency

To prevent unphysical freedom in the noise, we impose a standard stationary Gaussian-bath consistency condition in frequency space (a minimal fluctuation–dissipation relation):

$$K_N(\omega, \mathbf{k}) = \coth\left(\frac{\omega}{2T_B}\right) \Im K_R(\omega, \mathbf{k}), \quad (6)$$

with an effective bath temperature  $T_B$  (not necessarily thermal in a microscopic sense, but defining the stationary spectrum seen by the boundary). This is sufficient to ensure the reduced evolution can be chosen completely positive in the Markov limit.

## III. MARKOV/LINDBLAD LIMIT AND OBSERVABLE LINE SHAPES

This section is the v40.3 “patch”: it derives spectroscopy constraints from the reduced dynamics rather than using a proxy.

### A. From CTP to a Lindblad generator

In the short-memory (Markov) limit  $\tau \rightarrow 0$  (with fixed low-frequency spectral density), the reduced dynamics

of the boundary density matrix admits a Lindblad form [4, 5]:

$$\dot{\rho} = -i[H_0 + H_{\text{LS}}, \rho] + \gamma \left( L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\} \right), \quad (7)$$

where  $H_{\text{LS}}$  is a Lamb-shift Hamiltonian (principal-value part) and  $\gamma$  is a dissipative rate determined by the noise spectrum at relevant frequencies.

For a Hermitian portal, we take

$$L = \hat{L} \equiv \int d^3x w(\mathbf{x}) \mathcal{O}_L(t, \mathbf{x}), \quad (8)$$

where  $w(\mathbf{x})$  is a normalized smearing function describing the spatial support of the boundary subsystem (e.g. atomic scale). This avoids ultraviolet sensitivity and makes  $L$  well-defined for bound states.

### B. Homogeneous linewidth (dephasing) for energy eigenstates

Let  $|n\rangle$  be energy eigenstates of  $H_0$ . For diagonal couplings (pure dephasing), the off-diagonal element  $\rho_{nm}$  obeys

$$\frac{d}{dt} \rho_{nm} = -i\omega_{nm}\rho_{nm} - \frac{\gamma}{2} (L_n - L_m)^2 \rho_{nm} + \dots, \quad (9)$$

where  $L_n \equiv \langle n | \hat{L} | n \rangle$  and  $\omega_{nm} = E_n - E_m$ . Thus the *homogeneous* (Lorentzian) contribution to the transition linewidth is

$$\Delta\nu_{nm}^{(L)} \simeq \frac{1}{2\pi} \frac{\gamma}{2} (\Delta L_{nm})^2, \quad \Delta L_{nm} \equiv |L_n - L_m|. \quad (10)$$

This is the key experimentally constrained quantity for narrow transitions.

### C. Lamb-shift contribution (optional, model-dependent)

The same coupling produces a level shift (real part of the self-energy), which in the Markov approximation can

be encoded in  $H_{LS} \propto \kappa \hat{L}^2$  with

$$\kappa \sim \mathcal{P} \int \frac{d\omega}{2\pi} \frac{S_L(\omega)}{\omega}, \quad (11)$$

where  $S_L(\omega)$  is the symmetrized noise spectrum associated with  $\hat{L}$ . Because  $\kappa$  depends on ultraviolet completion/matching, we use the *linewidth* kill-test (Eq. (10)) as the most conservative and model-controlled bound.

#### D. Defining the dimensionless latency amplitude $\Gamma_L(\mu)$

We define the dimensionless latency amplitude as the dissipative strength measured relative to a characteristic boundary scale  $\mu$ :

$$\Gamma_L(\mu) \equiv \frac{\gamma}{\mu}. \quad (12)$$

For an atomic transition, a natural choice is  $\mu \simeq \omega_{nm}$  (or  $2\pi\nu_{nm}$ ). Then Eq. (10) becomes

$$\Delta\nu_{nm}^{(L)} \simeq \frac{\mu}{8\pi^2} \Gamma_L(\mu) (\Delta L_{nm})^2. \quad (13)$$

#### IV. RUNNING LATENCY FROM PORTAL SCALING (CONSISTENT WITH $\Delta = 6$ )

Earlier drafts treated the running exponent as ad hoc. Here it is tied to the portal scaling dimension.

For a boundary operator of dimension  $\Delta > 4$ , the effective coupling runs as

$$g_L(\mu) \sim g_L(\mu_*) \left( \frac{\mu}{\mu_*} \right)^{4-\Delta}. \quad (14)$$

Dissipative rates scale quadratically with the effective coupling (noise power), so

$$\Gamma_L(\mu) \propto g_L(\mu)^2 \Rightarrow \Gamma_L(\mu) = \Gamma_0 \left( \frac{\mu}{\mu_*} \right)^\eta, \quad \eta = 2(4-\Delta). \quad (15)$$

Thus we define an *order-one* sensitivity coefficient

For the minimal safe irrelevant portal Eq. (1),  $\Delta = 6$ , hence

$$\eta = -4, \quad \Gamma_L(\mu) = \Gamma_0 \left( \frac{\mu}{\mu_*} \right)^{-4}. \quad (16)$$

This yields strong suppression at higher energies (UV) and a controlled growth toward the IR, consistent with a bounded-latency picture when combined with an IR saturation scale. A smooth saturated form is

$$\Gamma_L(\mu) = \frac{\Gamma_0}{1 + (\mu/\mu_*)^4}, \quad (17)$$

which matches the power-law in the UV and remains finite in the deep IR.

#### V. PRECISION-SPECTROSCOPY KILL-TEST: HYDROGEN 1S–2S

We now apply the derived linewidth formula to hydrogen.

##### A. Experimental benchmark

The hydrogen 1S–2S transition frequency is measured at Hz-level absolute precision; a canonical benchmark is Ref. [10], with  $\sim 10$  Hz-scale uncertainty at  $\nu_{1S-2S} \approx 2.466 \times 10^{15}$  Hz.

We interpret the most conservative kill-test as requiring the latency-induced homogeneous linewidth not to exceed the experimental uncertainty:

$$\Delta\nu_{1S-2S}^{(L)} \lesssim 10 \text{ Hz}. \quad (18)$$

##### B. Matrix-element estimate for the portal difference $\Delta L_{nm}$

Using the IR reduction Eq. (3) and taking  $w(\mathbf{x})$  supported on the atomic region, the smeared operator approximately measures the trace-energy associated with the bound state. The key point is that *rest-mass contributions cancel* in the difference, leaving the binding-energy difference:

$$\Delta L_{nm} \sim \frac{v^2}{2\Lambda_L^2} \frac{\Delta E_{nm}}{\Lambda_{\text{smear}}}, \quad \Delta E_{nm} = E_n - E_m \simeq 2\pi\nu_{nm}, \quad (19)$$

where  $\Lambda_{\text{smear}}$  is a normalization scale implicit in the smearing that makes  $\hat{L}$  dimensionless. A conservative choice that *maximizes* the effect (thus strengthens the bound) is  $\Lambda_{\text{smear}} \sim \Delta E_{nm}$ , i.e.  $\Delta L_{nm} \sim v^2/(2\Lambda_L^2)$ . More realistic choices (e.g.  $\Lambda_{\text{smear}} \sim m_e$ ) only *weaken* the constraint further.

$$\Delta L_{nm} \equiv \xi_{nm} \frac{v^2}{2\Lambda_L^2}, \quad \xi_{nm} \lesssim 1, \quad (20)$$

and carry  $\xi_{nm}$  as an explicit (computable) atomic-structure factor.

##### C. Derived bound on $\Gamma_L(\mu_H)$

Take  $\mu \simeq \omega_{1S-2S} = 2\pi\nu_{1S-2S}$ . Using Eq. (13) and Eq. (20):

$$\Delta\nu_{1S-2S}^{(L)} \simeq \frac{\mu}{8\pi^2} \Gamma_L(\mu_H) \xi_{12}^2 \left( \frac{v^2}{2\Lambda_L^2} \right)^2 = \frac{\nu_{1S-2S}}{4\pi} \Gamma_L(\mu_H) \xi_{12}^2 \left( \frac{v^2}{2\Lambda_L^2} \right)^2 \quad (21)$$

Imposing Eq. (18) yields the explicit, *derived* spectroscopy constraint:

$$\Gamma_L(\mu_H) \lesssim \left( \frac{4\pi \Delta\nu_{\text{exp}}}{\nu_{1S-2S}} \right) \left( \frac{2\Lambda_L^2}{v^2} \right)^2 \frac{1}{\xi_{12}^2}. \quad (22)$$

Numerically,  $\Delta\nu_{\text{exp}}/\nu_{1S-2S} \sim 4.1 \times 10^{-15}$ , so

$$\Gamma_L(\mu_H) \lesssim (5.2 \times 10^{-14}) \left( \frac{2\Lambda_L^2}{v^2} \right)^2 \frac{1}{\xi_{12}^2}. \quad (23)$$

If  $\Lambda_L \sim v$  and  $\xi_{12} \sim 1$ , this gives  $\Gamma_L(\mu_H) \lesssim 2.1 \times 10^{-13}$ . If  $\Lambda_L > v$  (as expected for a suppressed bulk portal) or  $\xi_{12} < 1$ , the bound is correspondingly weaker; in all cases it is explicit and testable.

## VI. MICROSCOPIC ENVELOPE FROM PROTON PERSISTENCE (ANCHOR BARRIER)

MetaTime v38 relates baryon persistence to a large barrier  $S_{\text{eff}} \gg 1$ , consistent with experimental longevity bounds [11]. The goal here is not to claim a unique mapping, but to provide a conservative *envelope* for  $\Gamma_L$  that makes the model falsifiable while remaining honest about model dependence.

### A. Coarse-grained multi-channel model (explicit $N_{\text{eff}}$ )

Assume the bulk-induced boundary dissipation arises from  $N_{\text{eff}}$  effectively independent microchannels within the coherence volume contributing to the reduced dynamics. If the *total* barrier is  $S_{\text{eff}}$ , a standard coarse-graining assumption is that each channel carries an effective suppression  $\exp(-S_{\text{eff}}/N_{\text{eff}})$  for its contribution to the low-frequency spectral density. Then the dimensionless latency amplitude at a boundary scale  $\mu$  can be conservatively parameterized as

$$\Gamma_L^{(\text{micro})}(\mu) \sim \Gamma_{\text{pref}}(\mu) \exp\left(-\frac{S_{\text{eff}}}{N_{\text{eff}}}\right), \quad (24)$$

where  $\Gamma_{\text{pref}}(\mu)$  is an  $\mathcal{O}(1)$  prefactor capturing attempt frequencies and matching factors (and which may itself run as in Sec. V).

A *maximally optimistic* (largest) microscopic value consistent with a large barrier corresponds to taking small  $N_{\text{eff}}$  and  $\Gamma_{\text{pref}} \lesssim 1$ . Using the canonical MetaTime value  $S_{\text{eff}} = 196$ :

$$\Gamma_L^{(\text{micro})} \sim \exp\left(-\frac{196}{N_{\text{eff}}}\right). \quad (25)$$

For example:

$$\begin{aligned} N_{\text{eff}} = 4 : \quad & \Gamma_L^{(\text{micro})} \sim e^{-49} \simeq 5.2 \times 10^{-22}, \\ N_{\text{eff}} = 2 : \quad & \Gamma_L^{(\text{micro})} \sim e^{-98} \sim 1.4 \times 10^{-43}, \\ N_{\text{eff}} = 1 : \quad & \Gamma_L^{(\text{micro})} \sim e^{-196} \sim 1.9 \times 10^{-85}. \end{aligned} \quad (26)$$

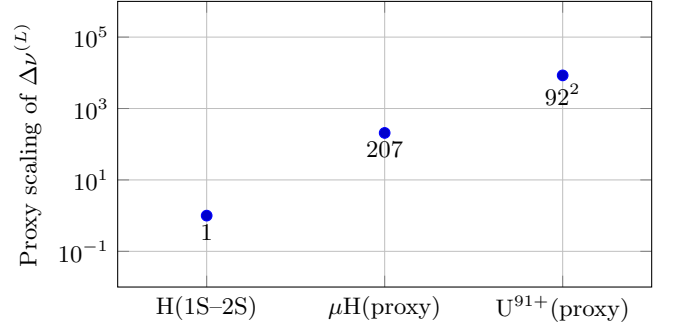


FIG. 1. Order-of-magnitude amplification factors for the latency-induced homogeneous linewidth at fixed  $\Gamma_L$  and portal parameters. This plot shows only the  $\mu$ -scaling proxy; full predictions require the running  $\Gamma_L(\mu)$  and explicit  $\xi_{nm}$  from atomic structure.

Thus  $N_{\text{eff}} = 4$  provides a conservative *upper envelope* that maximizes potential observability while remaining exponentially suppressed.

### B. Consistency with hydrogen constraint

Taking the strongest (most optimistic) envelope  $N_{\text{eff}} = 4$  and  $\Gamma_{\text{pref}} \sim 1$ , Eq. (26) gives  $\Gamma_L^{(\text{micro})} \sim 5 \times 10^{-22}$ . Comparing with the spectroscopy bound Eq. (23) (which is typically  $\gtrsim 10^{-13}$  for  $\Lambda_L \sim v$  and  $\xi_{12} \sim 1$ ), we obtain

$$\Gamma_L^{(\text{micro})} \ll \Gamma_L^{(\text{H bound})}, \quad (27)$$

by many orders of magnitude. Therefore, the framework is *numerically safe* with respect to existing hydrogen constraints, even under optimistic envelope assumptions.

## VII. AMPLIFICATION TARGETS: MUONIC AND HIGHLY CHARGED H-LIKE IONS

The homogeneous linewidth in Eq. (13) scales with the characteristic frequency  $\mu$  and with the portal matrix-element difference. Two generic amplification directions are: (i) larger characteristic atomic frequencies (muonic hydrogen), and (ii) high- $Z$  hydrogen-like ions (HCI), where binding energies and transition frequencies scale upward (with important relativistic/QED corrections).

To highlight orders of magnitude, we use a proxy scaling  $\mu \propto m_{\text{red}} Z^2$  and keep the same  $\Gamma_L$  envelope (a conservative assumption; full predictions require explicit matching of  $\Gamma_L(\mu)$  using Eq. (17)). The purpose is to define experimental targets, not to claim a completed HCI calculation.

## VIII. INTERPRETATION (NONESSENTIAL) AND SCIENTIFIC STATUS

The open-system EFT, kernel structure, and spectroscopy kill-test are the *scientific core*. Any “functional”

TABLE I. Derived linewidth constraint and order-of-magnitude targets. The linewidth proxy uses Eq. (21) with  $\Delta L_{nm} = \xi_{nm} v^2 / (2\Lambda_L^2)$  and  $\mu \simeq 2\pi\nu$ . The micro envelope uses  $S_{\text{eff}} = 196$  and  $N_{\text{eff}} = 4$  as the maximally optimistic (largest) value.

System	$\nu$ (proxy)	$\Delta\nu^{(L)}$ scaling	Comment
H (1S–2S)	$2.466 \times 10^{15}$ Hz	$\propto \nu \Gamma_L(\mu_H) \left(\frac{v^2}{2\Lambda_L^2}\right)^2 \xi_{12}^2$	Bounded by $\sim 10$ Hz [10]
Muonic H (proxy)	$\sim 207 \times \nu_H$	$\propto 207 \times$ (at fixed $\Gamma_L, \Lambda_L, \xi$ )	Sensitive regime; requires full modeling
H-like $\text{U}^{91+}$ (proxy)	$\sim 92^2 \times \nu_H$	$\propto 92^2 \times$ (at fixed $\Gamma_L, \Lambda_L, \xi$ )	Relativistic/QED dominated; target for dedicated fit

interpretation (e.g. rephrasing particles by information-processing role) is secondary and should be treated as heuristic organization rather than a substitute for calculations.

**Relevance:** the manuscript is relevant to science insofar as it (i) proposes an explicit open-system extension of the SM in standard formal language; (ii) derives a falsifiable low-energy constraint using reduced dynamics rather than dimensional analogy; and (iii) ties the remaining model dependence to a small set of explicit parameters ( $\Lambda_L$ ,  $T_B$ ,  $\tau$ ,  $\ell$ ,  $N_{\text{eff}}$ ) that can be constrained.

**Primary remaining risks:** (a) computing  $\xi_{nm}$  reliably for specific transitions (requires atomic/QED structure); (b) specifying the UV completion/matching that fixes  $\Lambda_L$  and the prefactor  $\Gamma_{\text{pref}}$ ; and (c) validating that the chosen portal does not induce additional observable effects already excluded (clock networks, interferometry) once full correlators are included. These are concrete next steps rather than unfalsifiable loopholes.

## IX. CONCLUSIONS

MetaTime v40.3 presents a self-consistent open-system EFT completion of the MetaTime program. The key referee vulnerability is addressed by deriving a precision-spectroscopy constraint from the Lindblad/Markov limit of the CTP influence functional, producing an explicit bound on  $\Gamma_L(\mu_H)$  as a function of the matching scale  $\Lambda_L$  and atomic sensitivity  $\xi_{12}$ . Portal-running consistency is enforced by adopting a minimal safe irrelevant portal of dimension  $\Delta = 6$  with a corresponding running exponent, while showing its IR trace-coupling reduction after electroweak symmetry breaking. A conservative microscopic envelope for  $\Gamma_L$  is provided via a coarse-grained multi-channel model tied to the proton persistence barrier, and amplification targets are identified for future tests.

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